

COMPARATIVE ANALYSIS BETWEEN GEOLOGICAL MODELLING METHODS

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RESUMO

Several stages of a mining enterprise depend on the existence of a geological model. Each of these steps is conducted by professionals who based on this model, perform tasks, and make decisions and plans. However, it is built from a sample dataset, and is therefore subject to uncertainties, which can be reduced with the accuracy of the data and the choice of a modelling method appropriate to each type of deposit. The purpose of this work was to analyze comparatively three methods for the elaboration of digital solid models, which involved manual, geostatistical and implicit modelling methodologies. The data used come from surveys of an iron ore deposit. The lithologies modeled were hematite and itabirite. The manual modelling was built through the parallel section method, which is based on interpretations that depend on the geological knowledge and the experience of the professional. However, the main problem that arises in this context is that many interpretations can be viable, making the method subjective. The elaboration of the geostatistical modelling was through the indicator kriging technique, which partly depends on interpretations, but the result is based on statistical principles, being less subjective than the manual method. Finally, implicit modelling, which uses radial-based functions, is performed “automatically,” so it does not offer the opportunity to add professional knowledge and experience. The results obtained in this research, using the mentioned methodologies, showed similarities in the determination of the models between the lithological contacts, but there were significant divergences in quantitative terms. Thus, it can be concluded that a fast automatic model can help in the interpretation or point out regions where more samples are needed, and that there may be interactivity between the professional who performs the interpretation of the sample data and the methodologies used, so that they can assist in the geological modelling process.

Keywords: geological modelling, indicator kriging, parallel sections, radial base functions.

1. Introduction

Geological modelling is the process used in the reconstruction of a mineral deposit and originates a three-dimensional virtual model, so that it establishes viable and facilitating conditions to know the behavior of the phenomenon under investigation, in order to make it qualitative and quantitative describable. A 3D modelling provides an overview of the mineral deposit as well as determining the extent, geometric pattern, location and spatial distribution of mineralization.

Mining enterprises have several peculiarities, which differ them from other industrial enterprises. Among them, we can highlight a very long maturation time. This time may take years or even decades, necessarily going through several steps, from surface mapping to production (Serra, 2000). One of the most complex stages of this process is the evaluation of deposits in which many studies are conducted for determining the viability of a mining enterprise. Generally, these studies are conducted by a multidisciplinary team, which depends on the existence of a geological model to elaborate technical, economic and environmental projects. This model is also fundamental in mine planning, so it is essential that it is as accurate and true as possible.

However, this model is built from a sparse sample data obtained from core boring. Generally, sampling campaigns are always planned with a cost / benefit ratio, which is, try to avoid oversampling errors that would lead to cost increase, and could make the implementation of the process unfeasible, as well as subsampling errors that could lead to increased estimation process uncertainty. Thus, the total sample volume is usually very small in relation to the deposit size, just enough to guarantee the representativeness of the variables under investigation. According to Yamamoto (2001), the entire sampling-based estimation process is subject to error.

As a result, building geological models requires interpolation to estimate variables where the attribute has not been sampled. Different interpolation methods are available; they can be either classic or computational. In classical methods data estimation is based on interpretation principles, whereas computational methods use mathematical functions in estimation (Yamamoto, 2001). Finally, the quality of this model will depend on the accuracy of the data and the choice of an appropriate estimation method for each type of deposit.

Currently, there are technological tools that can do modelling “automatically”, however, do not offer the opportunity to add the knowledge and experience of the professional.

The main objective of this work was to perform a comparative analysis between three geological modelling methods:

- *manual modelling*: using the parallel section method. The definition of ore body geometry is based on personal appreciation, and the result is influenced by the geological knowledge and experience of the interpreter. However, the main problem that arises in this context is that many interpretations can be viable, making the method subjective;
- *geostatistical modelling*: the indicator kriging technique was used in the interpolation of the lithological data. To use of this technique is necessary understanding and specialized judgment regarding the geological knowledge of the deposit, but the final result is based on statistical principles, being thus less subjective than the manual method;
- *Implicit modelling*: this method uses radial base functions in the interpolation process. The model is built from the definition of a continuous three-dimensional function, which represents the lithological distribution. In this case, the geological knowledge and the experience of the professional have little influence on the result.

All three methods were tested using drillhole data from an iron deposit located in the Quadrilátero Ferrífero region. Micromine software was used to model the mineralization related to hematite and itabirite bodies.

2 State of the Art

2.1 Geology of Quadrilátero Ferrífero

Quadrilátero Ferrífero (QF) is located in the south-central region of the state of Minas Gerais. Currently, the rocks of the region are grouped, due to common characteristics, into three main units: the granite-gneiss complex, Rio das Velhas

Supergroup, both of Archean eon, and Minas Supergroup, related to the Paleoproterozoic eon. In this way QF comprises several geological units formed at different times in the past and progressively got closer or overlapped geographically (Uhlein & Noce, 2012). In image 1, the geological map is presented, where the main lithologies that occur regionally can be observed.

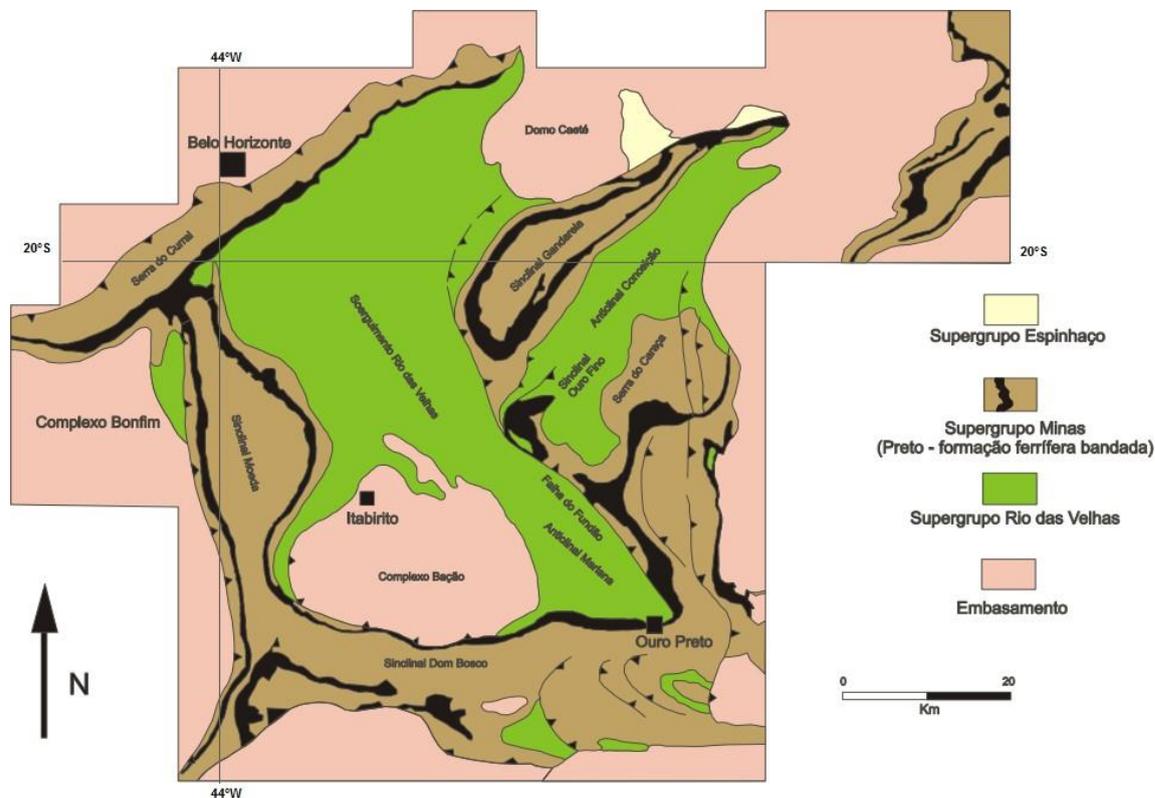


Image 1 - Simplified geological map of Quadrilátero Ferrífero. Source: Alkimim & Marshak (1998).

2.1.1 Granite-Gneiss Complex

It emerges in two regions in the Quadrilátero, in the center of the QF (Bação Complex) and surrounding the QF region. The granite-gneiss rocks are mainly made up of banded gneisses, of tonalitic to granodioritic composition that show migmatization features. Gneisses are rocks made up of minerals such as quartz, feldspar and biotite, arranged in alternating color bands, from light gray, rich in quartz and feldspar, to dark bands rich in biotite. These gneisses resulted from the metamorphism and deformation of granitic rocks that crystallized mostly around 3 billion years ago (Uhlein & Noce, 2012).

2.1.2 Rio das Velhas Supergroup

It consists of volcanic (mainly basalt) and sedimentary rocks. Subsequently, this group of volcano sedimentary rocks underwent an orogenic process, with metamorphism and folding. Thus, basalts and sediments were transformed into various types of schists (metamorphic rocks).

Rio das Velhas Supergroup comprises a basal unit, Nova Lima Group. This unit is composed of a combination of volcanic and sedimentary rocks, metamorphosed to a low degree (Uhlein & Noce, 2012).

2.1.3 Minas Supergroup

Minas Supergroup can be subdivided into three units: (basal clastic unit) Caraça Group, (intermediate chemical unit) Itabira Group and (top clastic unit) Piracicaba Group.

Itabira Group starts at Cauê Formation, composed of itabirites (banded iron ore formations). The itabirites are quartz or dolomitic and contain lenticular bodies of hematite (rich in iron ore). Gandarela Formation, at the top of the group, shows dolomitic marbles, subordinate itabirites and phyllites. Piracicaba Group basal unit, Cercadinho Formation, is characterized by the alternation of quartzites and phyllites, often ferruginous. Fecho do Funil Formation consists of quartz phyllites, dolomitic phyllites and dolomite lenses. Taboões (orthoquartzites) and Barreiro (graphite phyllites) formations have restricted occurrence (Uhlein & Noce, 2012).

At the top of Minas Supergroup is Sabará Group, consisting of chlorite-schists and phyllites, metagraywackes, metatuff, metaconglomerates and quartzites. Itacolomi Group, which covers Minas Supergroup, is restricted to an area south of Ouro Preto, where it forms the quartzite mountain with the same name (Uhlein & Noce, 2012). Espinhaço Supergroup overlaps Minas Supergroup and is composed of quartzites, metaconglomerates and phyllites (Takehara, 2004).

2.2 Manual modelling

Manual modelling is also known as the parallel section method. The geometry of a mineralized zone is illustrated in a series of vertical or horizontal sections that systematically intersect the ore body. The vertical sections usually coincide with the drillhole lines. The horizontal sections are generated at certain levels or dimensions; they are built based on the interpolation of the already interpreted vertical sections information. These sections are usually interpreted manually, using modelling software or even a drawing board, having the survey data and comparisons with genetically similar deposits in hands, the vertical and horizontal sections outlining mineralization and wall rocks are generated. The spatial union of these sections will result in a three-dimensional model of the ore body (Souza, 2007).

According to McLennan & Deutsch (2007), the parallel section method is an explicit method of geological modelling. Although the method is quite clear, there are important limitations, including significant time consumption, subjectivity rather than repeatability, inflexibility and inability to access the uncertainty limit.

Simple

Although the procedure is arduous, the explicit method of scanning polygons in various cross sections is definitely simple. In fact, this is the main reason for its popular use.

Long

The 2D contour lines and connecting lines drawing takes a huge amount of time. It is very common for a professional to spend up to three months developing a geological model.

Subjectivity and non-repeatability

The mineralization volume is essentially composed of prolonged series of small subjectivities and deterministic decisions, such as each corner of the contour line of each cross section that is chosen by the professional performing the modelling. Inevitably, an interpreter's signature is provided to the limit. For example, geologists, geophysicists, and

engineers can all consistently generate models, significantly different models with the same information. Thus, there is a need to generate objective contour surfaces.

Inflexibility

It is very difficult to update a geological model about the advent of new drillholes or new information. Typically, modifications are made to campaigns.

Unreachable Uncertainty

It is difficult to assess overall uncertainty at these geometric boundaries between sample data. This uncertainty can be a major source of insecurity in many situations. For example, in a gold deposit, the volume of mineralization is a vital economic indicator for project management. Ignoring volumetric uncertainty, considering only an explicit modelling boundary can devastate the venture.

Geologically realistic

Although all these limitations of the manual method, the resulting boundaries will be geologically realistic especially when interpreted by the same professional. There is direct control of this goal in the scanning process. There is no simple way to incorporate multiple possible contour realizations representing uncertainty. This is in fact the single most important limitation of the method.

2.3 Geostatistical Modelling

The geostatistical modelling process involves the estimation of spatial variability, which aims to detect areas where ore concentrations exist and can be delineated. The study of this spatial variability is performed by applying techniques on georeferenced data. Among the techniques used in the study of spatial variability; we highlight Geostatistics, which is based on the Theory of Regionalized Variables.

The basics of geostatistical methodology can be based on the following context: “The distribution of a given mineral asset within a deposit is mixed in nature, partly structured and partly random. On the one hand, the mineralization process has a global structure and follows some laws, geological or metallogenetic: in particular, rich and poor class zones always exist, and this is only possible if class variability has a certain degree of continuity. Depending on the type of ore deposit, this degree of continuity will be more or less demarcated but it will always exist. However, mineralization is never so chaotic that it excludes all forms of forecasting, nor is it sufficiently regular enough to allow the use of a deterministic forecasting technique” (Journel & Huijbregts, 1978). The geostatistical estimation process considers both the structured and random aspect inherent in any mineral deposit. Matheron proposed the name Geostatistics to designate a methodology that reconciles these two aspects (Journel & Huijbregts, 1978).

Geostatistics designates the statistical study of natural phenomena, often characterized by the spatial distribution of one or more variables, called regionalized variables (Journel & Huijbregts, 1978). Geostatistics has the objective the study and the structural representation of regionalized variable to solve estimation problems, based on measured experimental data through drillholes that do not fully cover such areas (Yamamoto Landim, 2013).

A regionalized variable, besides having the spatial continuity property, may have the following attributes: location, anisotropy, transition and zone of influence. In the study of these variables behavior there are two fundamental tools of geostatistical methods: variogram and kriging (Landim, 2003). The first geostatistical studies were started by Krige in the 1950s and conceptualized in Matheron's several studies in order to

provide accurate estimates of local levels of the mining blocks, eventually developing the formalization of geostatistical methodology (Yamamoto & Landim, 2013).

Variogram

Variogram is nothing more than a spatial correlation function that represents the basis for estimating spatial variability in geostatistics (Yamamoto & Landim, 2013). This is used to assess the spatial behavior of the regionalized variable, in terms of influence zone on a sample, anisotropy directions, and the degree of data continuity related to distance (Gringarten & Deutsch, 2001). Through variogram, it is possible to measure the degree of similarity between neighboring samples. Because it is expected that the closer these samples are collected, spatially or temporally, the greater the similarity between them is, and, therefore, the smaller the variance and the greater the spatial or temporal correlation, and the farther away the lower the resemblance will be, until these differences are attributed only randomly (Vilela, 2004).

According to Landim (2003), the variogram is built from a value range obtained from samples collected at regular intervals within the same geometric support. Being $x_1, x_2, x_i, \dots, x_n$, realizations of a regionalized variable and considering that the influence of each sample is greater the smaller the distance between the sampled points, it is necessary to define the distance vector Δh which has specific orientation. Thus, the variogram function is calculated as the average of the differences between points separated by a distance h , according to the formula:

$$\gamma(h) = \frac{1}{2N} \sum_{i=1}^n (Z(x) - Z(x+h))^2 \quad (2.1)$$

Where $\gamma(h)$ is the variance of data pairs separated by distance h , (h) , is the distance between the data that forms a pair, $Z(x)$ he value of the variable in position x , the value of the variable separated by the vector h and N is the number of pairs found for each distance h .

With the construction of a graph confronting variance versus distance, the experimental variogram is obtained (Image 2), which needs to be adjusted to a theoretical model. This adjustment is called variographic modelling and consists of defining a mathematical function that represents the behavior of the data as distance function for any distance h . The most used mathematical models are exponential, spherical and Gaussian (Souza, 2007).

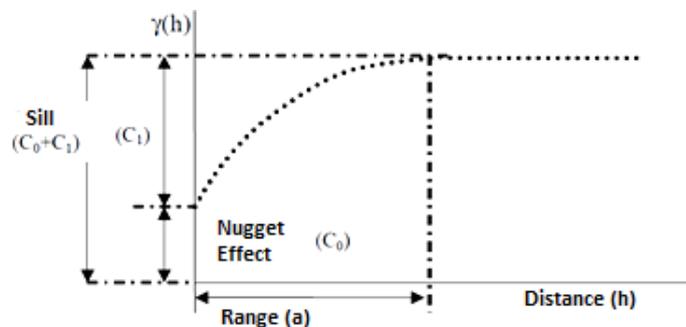


Image 2 - Typical variogram model Vilela (2004).

The difference between the values $(z_{(xi)} - z_{(xi+h)})^2$, average, is increasing with the distance to a certain point, from which it stabilizes at a value called sill (C_0+C_1) and is approximately equal to the data variance. The level indicates the value by which the function stabilizes in the random field, corresponding to the distance “a” called range, and shows the maximum variability between pairs of values (Landim, 2001). The source discontinuity is called the Nugget Effect (C_0) and as Journel & Huijbregts (1978) would be due either to measurement errors or to microvariability of mineralization.

The study is done in one direction along one line or along a series of parallel lines, using n possible differences at intervals h or multiples of h. In addition to distance and direction, additional parameters need to be defined, for each data point, a window is determined, within which there can be one or more points, or none at all. This window is defined by direction, angular tolerance and maximum width, as well as step size (distance) and step tolerance. The maximum width parameter aims to limit the indefinite opening of the search window given by angular tolerance (Yamamoto & Landim, 2013).

Variographic modelling is the first step in applying kriging, and it is the parameters obtained by variogram that control the distribution of Kriging weights, which will influence the interpolation of data (Souza, 2007).

Kriging

Kriging is the geostatistical process of estimating the values of variables distributed in space and / or time, based on close values when considered interdependent by variogram. When compared with traditional estimation methods, only kriging presents unbiased estimates and the minimum variance associated with the estimated value. Kriging is named after mining engineer Daniel G. Krige who pioneered the application of statistical techniques in mining assessment (Yamamoto & Landim, 2013).

Kriging provides, in addition to the estimated values, the error associated with such an estimate. It is an estimation method based on a series of regression analysis techniques, either linear or nonlinear transformations. Thus, kriging techniques can be divided into linear and nonlinear. One technique is said to be linear when the continuous variable is kept at the original scale such as simple, average, and ordinary kriging; and nonlinear when nonlinear data transformation occurs, such as multigaussian, lognormal and indicator kriging (Yamamoto & Landim, 2013).

Data transformation is required under various circumstances for geostatistical estimation. Although these transformations change the values of average and variance, the goal is to modify the shape of the frequency distribution. Transformations are made by means of a mathematical function that assigns each value x a new value f (x) (Yamamoto & Landim, 2013).

$$Y = f(x) \quad (2.2)$$

Indicator Kriging

Indicator kriging is a nonlinear interpolation method that consists of applying Ordinary Kriging over a transformed variable, that is, the variable resulting from the application of a dichotomy indicator function (Landim and Sturaro, 2002).

Ordinary kriging is a widely used local estimation method for the simplicity of the results it provides. Where the estimation at an unsampled point is made by the linear combination of values found in the near vicinity (Yamamoto & Landim, 2013). The ordinary kriging estimator is:

$$Z^*(x_0) = \sum_{i=1}^n \lambda_i Z(x_i) \quad (2.3)$$

The first step in the indicator kriging is the transformation of the original data into indicators, any type of variable becomes a categorical variable 0 or 1.

The methods that use the concept of indicators are an appropriate alternative for lithology modelling, as they are categorical variables measured at nominal scale. This is also a well-accepted technique for estimating extreme or biased values (Souza, 2007).

According to Yamamoto & Landim (2013), a continuous random variable can be discretized in relation to a reference value, cutoff content. Categorical variables, measured at nominal or ordinal scale, have a discrete number of types and can be discretized into k types by binary coding as follows:

$$I(x, k) = \begin{cases} 0, & \text{se } Z(x) \neq \text{tipo } k \\ 1, & \text{se } Z(x) = \text{tipo } k \end{cases}$$

Kriging variances, being conditioned only by the geometric arrangement of the points and therefore independent of the sample values, are usually not measures that allow the degree of precision of the local estimate to be established. To fulfill this need one of the pointed out solutions is the indicator kriging. In this case, the focus is not to estimate a certain value, as in ordinary kriging, but to define areas that are more or less likely to occur (Landim & Sturaro, 2002).

2.4 Implicit Modelling

Implicit modelling is a method of mathematically fitting a surface to a series of known data points. This adjustment process involves the interpolating of this surface between these points. One way to do this is to get function values at some discrete points in the gap of interest. Then a simpler function can be developed to adjust this data. This application is known as curve fitting (Bertolani, 2010).

Among the advantages of implicit methods, we can highlight the lower noise sensitivity and the ease of combining implicit functions for modelling purposes. However, the same features that make implicit methods robust against noisy and incomplete data can produce undesirable smoothing on surfaces. In addition, it is often difficult to achieve local control over distance functions (Polizelli, 2008).

The implicit modelling technique used in this work makes use of radial base functions (RBF) as an interpolation method.

Radial Base Functions (RBF)

RBFs are functions which value depends solely on the radial distance between a base point and a field point, which may also be another base point. Some properties that guarantee the potentiality of the RBFs are evident: the angular symmetry is the most important, since the value of the function depends only on the distance from the argument to the base point and any rotations do not influence its result. Another important factor is that their values can always be positive. However, other factors may become equally interesting: often decay and smoothness characteristics may be appropriate, and in this sense, there are several classes of functions that can be adjusted to these requirements. The interpolation method with radial functions is one of the techniques used in multidimensional interpolation, which become interesting options to develop due to its ease of implementation, fast computational processing and results accuracy (Bertolani et al., 2010).

The problem of surface representation or reconstruction can be expressed as follows:

Data n distinct points $\{(x_i, y_i, z_i)\}_{i=1}^n$ on a surface M in \mathcal{R}^3 , find a surface M' that is a reasonable approximation M (Carr *et al*, 2001).

Definition: according to Bertolani (2010), a function $\varphi: \mathcal{R}^n \rightarrow \mathcal{R}$ is called radial as long as there is a single argument function, $\varphi: [0, \infty] \rightarrow \mathcal{R}$ such that

$$\varphi(x) = \varphi(r) \tag{2.4}$$

where $x \in \mathcal{R}^n$ and $\|x\|$ is the Euclidean norm of space \mathcal{R}^n .

To simplify the notation is called $r = \|x - c\|$, which interpretation is the distance from the point defined by the vector o x to the point c and by convention $r = \|x\|$, there is an infinite number of radial base functions, only a few are commonly used in different applications. The radial base functions that are most used in the interpolation method are called kernels, and below Table 1 presents the main kernels (Wright, 2003).

Table 1- Most common radial base functions.

Function	Equation
Gaussian (G)	$\varphi(r_i) = e^{-c^2 r_i^2}$
Thin Plate Splines (TPS)	$\varphi(r_i) = r_i^4 \log(r_i)$
Cubic (C)	$\varphi(r_i) = r_i^3$
Multi-Quádric (MQ)	$\varphi(r_i) = \sqrt{c^2 + r_i^2}$

RBF as interpolator method

Interpolation with the use of radial base functions focuses, from known data, the reproduction of unknown functions (Bertolani, 2010). Using proper radial base functions, the interpolation function can be written like this (Turk & O'Brien, 2002):

$$f(x) = \sum_{i=1}^n \lambda_j \varphi(\|x - c\|) + P(x) \tag{2.5}$$

where n is the sample size, λ_j are weights and $P(x)$ is a polynomial of degree m .

A restriction placed on the coefficients is:

$$\sum_{i=0}^N \lambda_j P(x_i) = 0 \tag{2.6}$$

As $\{p_1, \dots, p_l\}$ a base of polynomial p_i maximum degree m and $\{c_1, \dots, c_l\}$ the coefficients that describe P in the base, equations 2.4 and 2.5 can be written through the linear system

$$\begin{pmatrix} A & P \\ P^T & 0 \end{pmatrix} \begin{pmatrix} \lambda \\ c \end{pmatrix} = \begin{pmatrix} f \\ 0 \end{pmatrix} \tag{2.7}$$

where $A_{x,c} = \varphi(\|x - c\|)$, $P_{x,c} = p_c(x)$, and f is the value of function to be determined at points of the sample.

As all points of sample S are located on the surface and therefore present $f(x_i) = 0$, new points need to be added to the system to avoid trivial solution $\lambda_i = 0$ for any i . These points are generated from each point x_i moving in the direction of the normal vector (Turk & O'Brien, 2002).

Off-surface points are assigned positive values, while inside points are assigned negative values (Carr et al, 2001).

In the geological modelling process, according to Cowam *et al* (2003), it is necessary to define one of the lithologies to represent positive values of the function and others to represent negative values. Then, lithology data sampled by drillholes is converted from the lithological code to numerical values. Thus, a value of zero is assigned to the points located between the contact surfaces. Which can now be treated as a scattered data interpolation problem. Since the data is interpolated at the places where the points are zero, a function can be extracted from the contact surface between the two lithologies.

Also according to this same author, the RBF interpolation process is similar to the interpolation process using kriging. The difference between kriging and RBF interpolation is that kriging uses the covariance function obtained from the data (the variogram) and RBF uses a basic function that is chosen from a standard range of functions.

3 Materials and Methods

3.1 Database

To test the proposed methodologies, we used a database that has 254 drillholes that originated 2,171 samples, and data resulting from chemical analysis and geological description. The holes are vertical with depths ranging from 4 to 334 meters distributed in semi-regular mesh in the center and irregular in the periphery. The average sample size along the hole is 10 meters. Image 3 shows the drillhole location map.

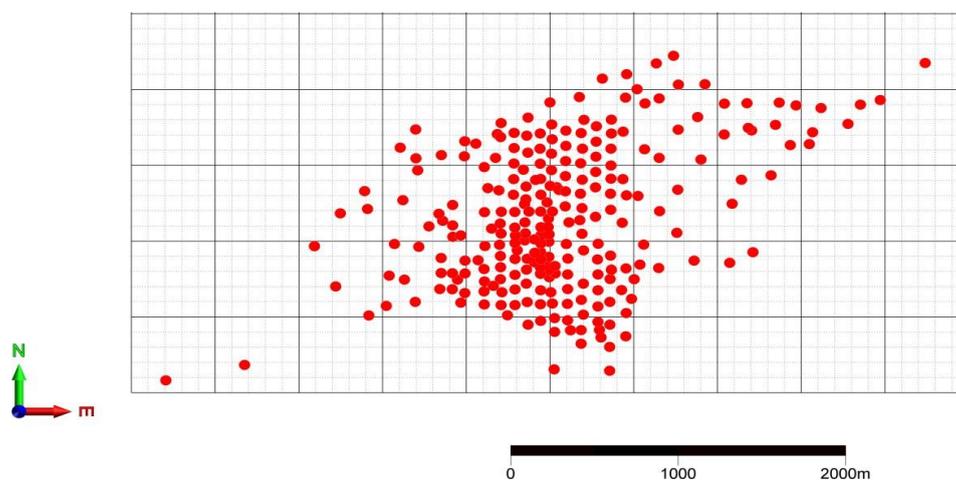


Image 3- Sample location map.

Three lithological classes are described, hematite, itabirite and yoke. Only data referring to the two modeled classes I (itabirite) and H (hematite) were selected.

Geological modelling was performed using Micromine, a modular software for exploration, 3D geological modeling, resource estimation, mine planning and design, pit optimization and resource allocation.

With the Micromine software input database ready, they were imported and validated by the program, and the deposit modeling process began.

3.2 Manual Modelling

The domain area of the samples was divided into vertical sections along the N-S direction trying to match the drillhole lines, totaling 30 sections with line spacing ranging from 20 to 50 meters in the most central region. The set of holes with their respective lithologies represented in each profile constituted a section, thus allowing the visualization of the lithological contacts. Each lithology was designed respecting the spatial distribution of the samples (Figure 4).

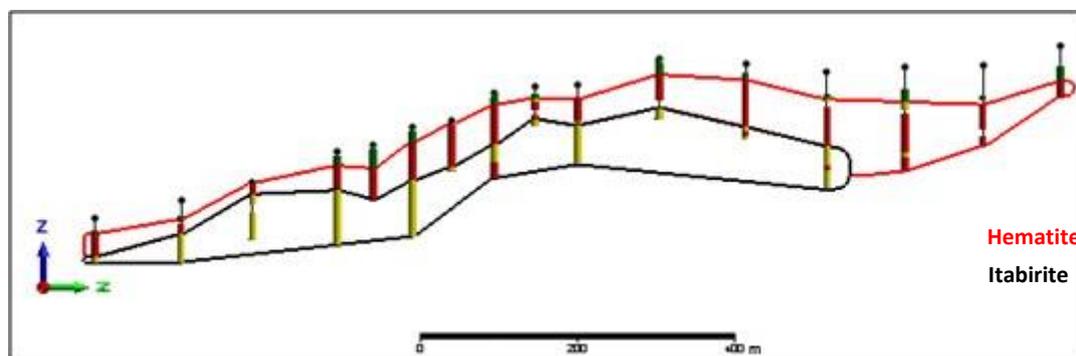


Image 4 - Vertical section 11 containing their respective holes and geological interpretations.

The horizontal sections were generated from dimension 600 to dimension 1020, and the interpretation of these sections was based on the envelopes of the vertical sections already interpreted. To facilitate modelling and avoid “empty” spaces between lithological domains, the sections were overlapped drawn (Image 5). The spatial union of these sections resulted in two solids, consequently with intersections, which were eliminated using the hematite model to cut the itabirite model.

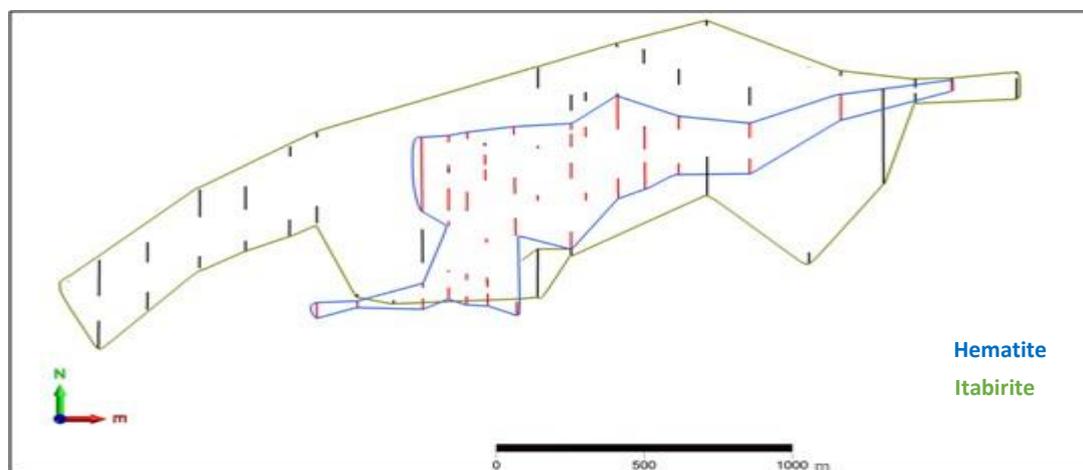


Figure 5- Interpreted horizontal section dimension 820.

3.3 Geostatistical Modelling

All samples corresponding to the itabirite lithology, received the value 1 as an indicator, meaning that in the samples located at these points the probability of occurrence of itabirite is 100%. The other samples corresponding to hematite and yoke received the value 0 as an indicator, that is, the probability that they are present at these points is null, and they are treated without any distinction at this stage in which the itabirite is being modeled. The same procedure is adopted for mineralization of hematite bodies.

The variograms were generated using Micromine software, the first one calculated along the hole to define the nugget effect, considering that the shortest distance between the samples is along the hole. The parameters for variogram calculations were the same for both lithologies, being the number of lag equal to 10, lag equal to 100, band size 50 and angular tolerance 10°. The continuities of the variables were analyzed in eight directions, that is, from 0° to 157.5°.

The most continuous directions for hematite and itabirite mineralization are shown in Images 6 and 7.

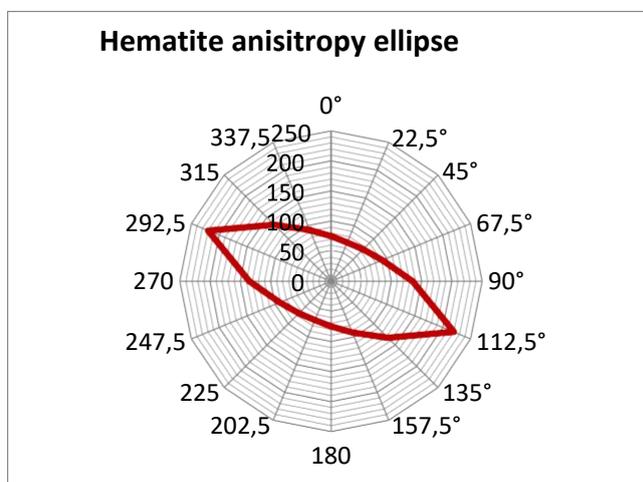


Image 7 - Continuity analysis graph for hematite mineralization.

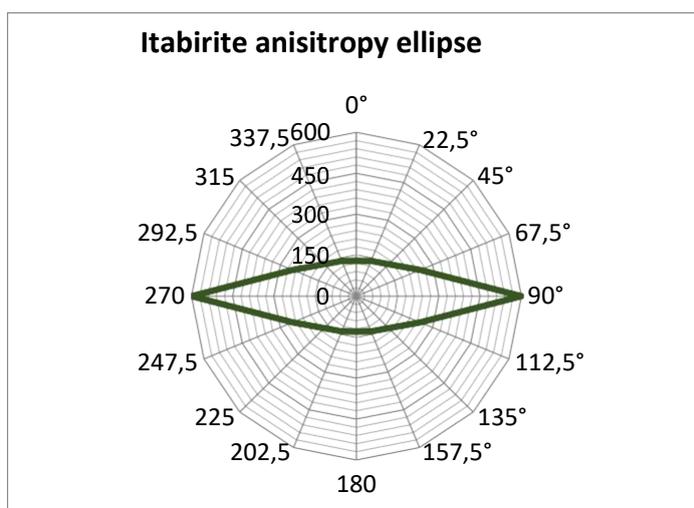


Image 7 - Continuity analysis graph for itabirite lithology.

Like any interpolating method, the kriging method generally uses a point grid or a block model, which is filled with the estimated values. In this study, a block model with dimensions 50x50x10 meters was used.

In addition to the control over the spatial behavior of values given by variographic modelling, there is also, the control over how data used to estimate each grid point is selected. This control is called the search strategy, where some parameters define how much data will be used, how far away from the estimated point the search will be made and even in which direction this search radius will be larger or smaller (anisotropy). Table 2 presents the search parameters used in the indicator kriging by Micromine software.

Tabela 2- Search strategy.

Search Strategy		
Discretization	x	2
	y	2
	z	2
Elipsoid search	Hematite	Itabirite
Octante	maximum 8	maximum 8
Radius	220	600
Azimuth	112,5°	90°
Spatial continuity	Hematite	Itabirite
Nugget effect	0,07	0,06
Partial sill	0,14	0,18
Model	Exponential	Exponential
Maximum range	220	600
Azimutheh	112,5°	90°

Another important fact that should be pointed out, to avoid extrapolation beyond the domain area of the samples, a solid was generated respecting these domains, the same was used to cut the models generated by the implicit and geostatistical method (Image 8).

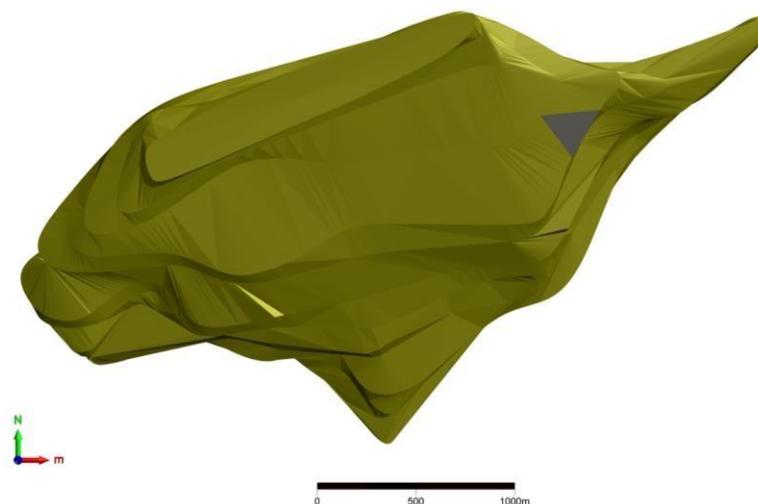


Image 8 – Extrapolation restriction model.

3.4 Implicit Modelling

In applying this method, it was necessary to establish the search parameters used by the algorithm (Table 3).

Table 3 - Search parameters used by the algorithm

Search Parameters	Values Used
Minimum size of skipped ranges	1
Maximum points per partition	500
Maximum space	30
Distance buffer	0
Triangular mesh grid units	50

Minimum size of ignored intervals: defines the minimum sample length interval that should be included in the lithology model, and which should be ignored.

Maximum points per partition: the data set is divided into overlapping regions (partitions), which are modeled separately for the sake of algorithm efficiency. This parameter defines the maximum amount of points to contain in a partition.

Maximum spacing: determines the maximum spacing (in grid units) that will be used to position the points relative to each range. It defines the spacing of each grid cell that will be used to position the points relative to each gap.

Distance buffering: this option allows you to “fill in” the input data boundaries in all directions according to a defined distance.

Mesh size: it determines the mesh size (in grid units) that will determine the size of the individual wireframe triangles.

You can also define a custom set of x, y, z coordinates to control extrapolation.

However, in the generation of this model, the same model presented in Image 8 was used.

4 Results and discussion

The manual model resulted from the spatial union of the horizontal sections. The application of the geostatistical method using the indicator kriging technique resulted in blocks filled with probability values, i.e., the probability of the estimated block belonging to the modeled lithology, blocks with values below 0.5 (50%) were disregarded. And from this set of selected blocks, the solids representing the geological models of hematite and itabirite were generated. Finally, the implicit model was generated after adjusting the settings of the algorithm search parameters. The envelopes corresponding to the mineralized bodies can be seen in Images 9, 10 and 11.

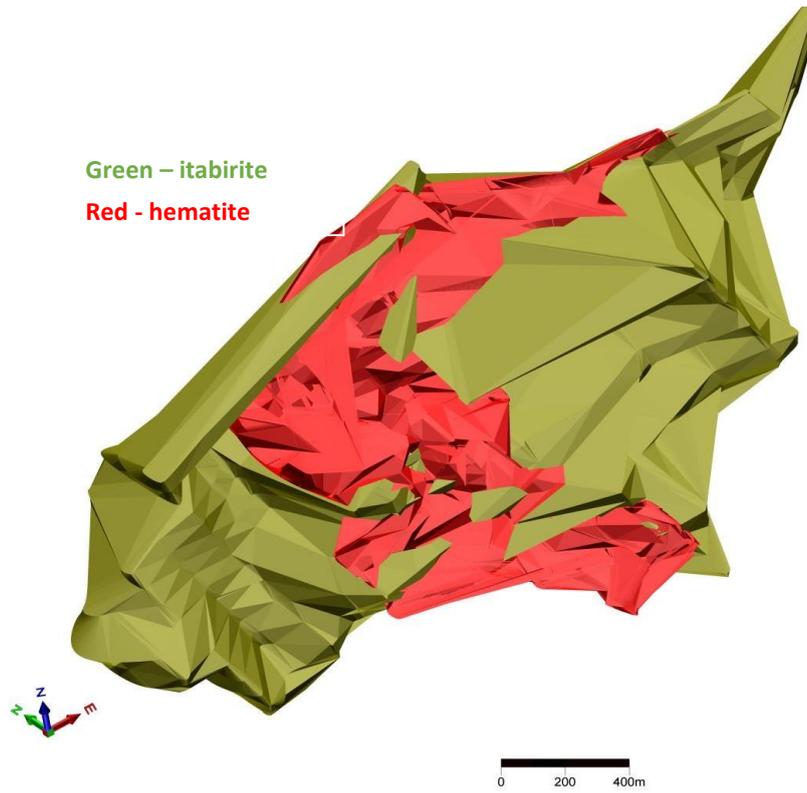


Image 9- Model Manual.

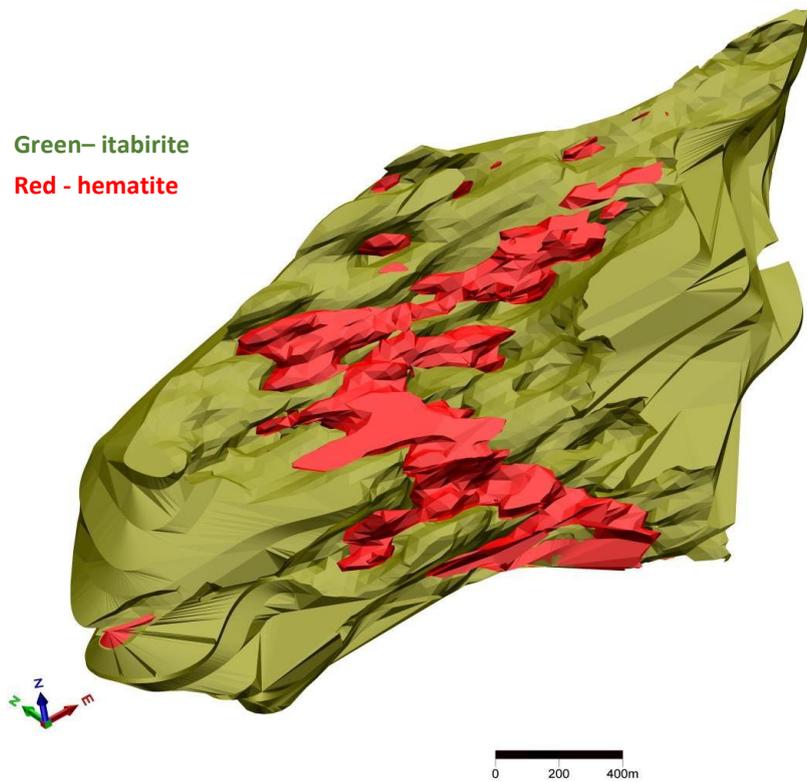


Image 10 – Model Geoestatistical.

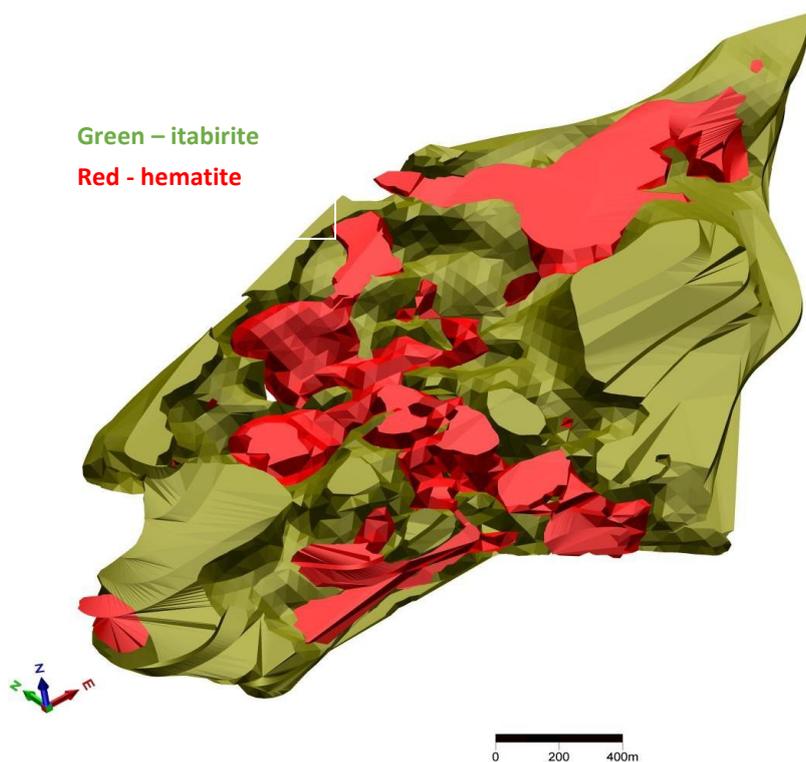


Figura 11 - Model Implicit.

4.1 Comparison of Results - Hematite body

Regarding the models generated for hematite lithology, through the implicit and geostatistical modelling methods, they presented some similarity in the distribution of the mineralization than when compared to the manual modelling method. Being visible the difference in the shape and continuity of the bodies (Image 12).

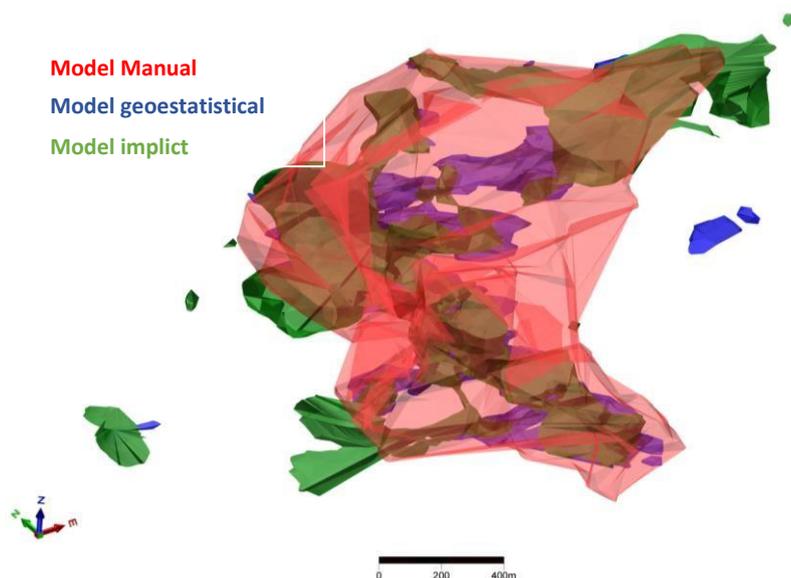


Image12 - Overlapping models of hematite mineralization..

Partly these differences were due to the fact that, where there were few samples representing hematite mineralization, with a larger predominance of samples representing

itabirite lithology, they eventually dominated the result. Thus, hematite samples were considered as itabirite in the implicit and geostatistical modelling process. In the manual modelling process, the interpretations of the vertical sections of this model generated envelopes based on the main hematite body mineralization, thus involving the small mineralization of the itabirite lithology, resulting in a more continuous solid and larger volumetric increment in relation to the implicit and geostatistical methods. This volume increase is best described in Table 4.

Table 4 - Solids volumes corresponding to hematite and itabirite lithologies.

	Hematite	Itabirite	
Models	Volume (m ³)	Volume (m ³)	Volume Total (m ³)
Manual	138.547.206	436.031.462	574.578.668
Geoestatistical	40.757.820	644.183.377	684.941.197
Implict	71.582.274	567.504.108	639.086.382

4.2 Comparison of Results - Itabirite body

Regarding the manual model generated for itabirite lithology. In the interpretation of the sections, we tried to extrapolate as little as possible beyond the domain area of the samples, which resulted in a model with smaller geometry when compared with implicit and geostatistical modelling methods. These methods showed greater volume increase because hematite samples were considered as itabirite, in places where itabirite samples prevailed (Image 13). These differences interfered with the results of the volume values of each model, as can be seen in Table 5 above.

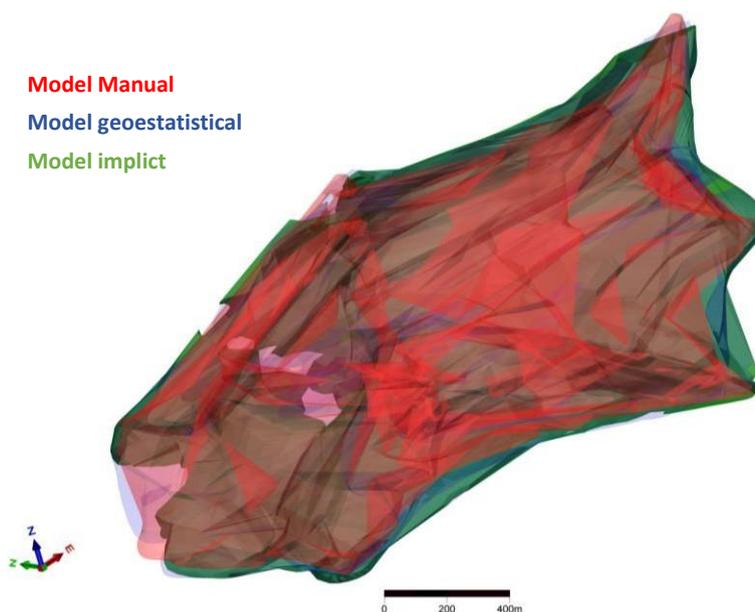


Image13- Overlapping models of itabirite litology.

5 Conclusion

The results obtained with the use of the three methods in this research showed similarities in the determination of the models between the lithological contacts, but there were significant differences in quantitative terms.

The geostatistical and implicit modelling methods presented similar results, mainly in the hematite body modelling. Such methodologies proved to be applicable as auxiliary tools to manual modelling in the process of section interpretation. Thus allowing a continuous improvement of the model making it more refined and reliable.

The implicit modelling method generates models automatically and quickly that can help in interpreting or pointing regions where more samples are needed, much more flexible when compared to manual and geostatistical modelling methods.

Thus, it was concluded that there might be interactivity between the professional who performs the interpretation of the sample data and the methodologies used in this article.

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